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No. 307

STRENGTH OF TUBING UNDER COMBINED AXIAL  
AND TRANSVERSE LOADING

By L. B. Tuckerman, S. N. Petrenko, and C. D. Johnson  
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For the past two years the Bureau of Standards, in cooperation with the Bureau of Aeronautics of the Navy Department, and with the National Advisory Committee for Aeronautics, has been carrying out a systematic study of the strength of duralumin and chrome-molybdenum steel round tubing in combined transverse and axial loading.

The program of tests as originally planned covered the following variables:

Material: Duralumin tubes complying with Navy Department Specification No. 44-A-2, October 1, 1926\* and Alloy Steel tubes complying with U.S. Army Air Service Specification No. 10231-B, June 21, 1926.\*\*

Diameter of tubes: 1, 1-1/2 and 2 inches outside diameter.

Thickness of wall: From about 1/70 to about 1/5 the outside diameter.

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\*This material also complies with Army Navy Specification No. AN9092 (1929 issue).

\*\*Identical with U.S. Army Air Service Specification No. 57-180-2, December 8, 1926.

Length of specimen:

For each of the three diameters and for each thickness of wall the 1, 15, 30, 50, 75, 100, and 120 slenderness  $\left(\frac{l}{r}\right)$  ratios.

Loading conditions and observations made during the test:

In the transverse test the specimens were supported as simple beams and loaded at two points each one-third the span from the reactions. Both the load and the deflection were recorded.

In column tests the specimens were loaded by axial compressive loads using spherical loading blocks so that the specimen was a "round end" column. The loads and the deflections at right angles were recorded.

In the combined column and transverse tests the specimens were loaded transversely as for the transverse tests with loads which were a given fraction  $m$  of the ultimate transverse load previously determined. This fraction of the ultimate transverse load was  $m = 20, 40, 60, \text{ and } 80$  per cent.

Axial compressive loads were applied as for the column tests until failure occurred.

The maximum axial load and the corresponding deflection of the specimen were recorded.

Number of specimens:

Two duplicate specimens for each loading condition.

The tests so far carried out have covered duralumin tubes 1-1/2 in. outside diameter with wall thicknesses 0.032, 0.049,

0.058, and 0.072 in.; and chrome-molybdenum steel tubes 1 in. outside diameter with wall thicknesses 0.035, 0.049, and 0.188 in.\*; 1-1/2 in. outside diameter with wall thicknesses 0.049, 0.065, 0.083, and 0.281 in.\*; and 2 in. outside diameter with wall thicknesses 0.065, 0.083, and 0.095 in.

These data have been studied in many ways in an effort to draw from them conclusions of general validity, and by a combination of theoretical and empirical reasoning it has been found possible to combine them into a form both suitable for practical use and adapted to further study.

#### B a s i s   o f   S t u d y

Axial load: A previous unpublished study of column action based on the Karman-Engesser double modulus theory had shown that, in cases where secondary or detail failure did not occur, the data on column tests were best compared by the introduction of two new variables:

$$\lambda = \frac{\frac{l}{r}}{\pi \sqrt{\frac{E}{F_c}}} \quad \text{and} \quad \sigma = \frac{f_c}{F_c}$$

where  $\frac{l}{r}$  is the slenderness ratio of the equivalent "round end" column,

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\*The tests on 0.188 in. and 0.281 in. wall thickness are not completed and the experimental data on these tubes were not included in the attached diagrams.

$E$  = Young's modulus for the material  
(30,000,000 lb./sq.in. for steel,  
10,000,000 lb./sq.in. for duralumin),  
 $f_c$  = the column strength, i.e.  $\frac{\text{axial load}}{\text{sectional area}}$ ,  
 $F_c$  = the limiting value of the column strength  
for low values of  $\frac{l}{r}$  before pick-up  
occurs.

For materials with a well-marked yield point  $F_c$  is known to be practically identical with the yield point of the material in tension when tested at slow speeds.

By the use of these variables, columns made of materials of markedly different physical properties could be directly compared with each other. For materials whose stress-strain curves are affine curves, identical  $\lambda$ ,  $\sigma$  curves should be expected, although their yield point and modulus of elasticity differ widely.

In none of the tubes tested was there any indication that secondary or detail failure affected the measured loads so that it seemed reasonable to apply this analysis to the data.

On plotting the results of all of the pure column tests of chrome-molybdenum steel tubing on this basis (Figure 1), it was found that all of the points grouped closely around a single curve except in the neighborhood of  $\lambda = 1$ , where from theoretical considerations as well as experimental data the widest scatter, caused by the unavoidable variations in the material or

unavoidable small excentricities in the tubes, was to be expected. The agreement was much closer than between the curves for  $\frac{l}{r}$  and  $f_c$  (Figure 3).

Plotting on the same basis (Figure 2) the results for pure column load on the duralumin tubing, the curve coincides, as should be expected, with that for chrome-molybdenum tubing for values of  $\lambda \neq 1.4$  (Euler range) but falls noticeably below it in the neighborhood of  $\lambda = 1$ . This difference might have been anticipated because of the decidedly different shape of the stress-strain curves of the two materials and it excludes the possibility of making a single series of curves of this type serve for both the duralumin and chrome-molybdenum tubing.

#### Transverse loading - Modulus of rupture

No clear relationship between the shape of specimen, stress-strain curve of the material and the modulus of rupture has as yet been found. It has long been known that for identically shaped specimens of ductile materials, with similar stress-strain curves, the modulus of rupture is closely correlated with the tensile strength, so that a linear correction of the modulus of rupture for tensile strength over a limited range of tensile strength gives much more concordant results. The known correlation between tensile strength and indentation (Brinell, Rockwell, Vickers) numbers has therefore been used in studying these tests to correct for unavoidable differences in the material.

The results show that the ratio of the modulus of rupture to the tensile strength depends in a complex manner upon the  $\frac{d}{t}$  ratio and the slenderness  $\left(\frac{l}{r}\right)$  ratio. For each material and each diameter of tube, trends can be observed, but these cannot as yet be generalized to cover other materials and other thicknesses. For instance, for 2-inch chrome-molybdenum steel tubing the modulus of rupture decreases with increasing slenderness ratio, the rate of decrease being greater the greater the wall thickness. It also decreases with increasing wall thickness, the rate of decrease being greater the greater the slenderness ratio.

For 1-1/2 inch duralumin tubing at low slenderness ratios the modulus of rupture is higher for the thicker walled tubing and at high slenderness ratios lower. For the thinnest walls tested (0.032 in.,  $\frac{d}{t} = 47$ ) the modulus of rupture changes but little, increasing slightly with increasing slenderness ratio.

For all the chrome-molybdenum tubing the modulus of rupture varies from 20% to 40% higher than the tensile strength, being over 26% higher for all except the two-inch tubing at high slenderness ratios. In marked contrast, the modulus of rupture for the 1-1/2 in. duralumin tubing is nearly equal to the tensile strength.

Until some definite relationship based on experimentally verified theoretical grounds or based purely empirically on a more comprehensive series of tests is found, it has been thought

best to ignore the variation in the modulus of rupture with slenderness ratio and  $\frac{d}{t}$  ratio and use a safe constant value for the ratio between modulus of rupture and tensile strength.

For chrome-molybdenum tubing the modulus of rupture was assumed 26.3% higher than the tensile strength. This underestimates by about 12% the transverse strength of thin-walled short tubes but no higher value seemed safe to use generally since the value chosen overestimates by about 8% the transverse strength of the 2 in., .095 in. walled tubing at slenderness ratios of 75 and over.

For duralumin tubing 1-1/2 in. in diameter the modulus of rupture was assumed equal to the tensile strength. This overestimates by about 8% the strength of light-walled short tubes. With properly chosen factors of safety these overestimates which are well within the experimental errors should not be considered dangerous.

#### Combined transverse and axial loading

Lacking any satisfactory theoretical basis upon which to analyze these tests, a number of empirical methods for combining them have been tried. Finally, the experiment was made of reducing the axial stresses and the slenderness ratios upon the same basis as under pure axial loading, leaving the stresses due to transverse loading unchanged. For each of the transverse loads ( $m = 20, 40, 60$ , and  $80$  where  $m$  is the ratio of the maximum bending stress  $f_b$  to the modulus of rupture  $R$  in



per cent) the axial stress  $f_c$ , was computed from the axial load at failure and the value  $\sigma = \frac{f_c}{F_c}$  computed with the value of  $F_c$  determined from the tests under purely axial loading. The slenderness ratios were reduced to values of

$$\lambda = \frac{1}{\pi \sqrt{\frac{E}{F_c}}} \frac{l}{r}.$$

The resulting curves are shown in Figures 1 and 2. It will be noticed that points having the same slenderness ratio are scattered over different values of  $\lambda$  in such a manner as to lie much more closely to a single curve than when the axial stress  $f_c$  is plotted against the slenderness ratio  $\frac{l}{r}$  (Figures 3, 4, and 5). This is particularly noticeable on the chrome-molybdenum steel curves where the values of  $F_c$  differed markedly.

Considering the difficulty of the tests, the large variation in physical properties of the tubes and their unavoidable deviations from their nominal dimensions, these points representing tests on tubing of eight different sizes ranging from 49,900 to 63,500 lb./sq.in. in tensile strength for duralumin and from 97,700 to 150,500 lb./sq.in. for chrome-molybdenum steel fall on the average curves as closely as could be expected. This method which has a theoretical basis only for purely axial loads is therefore seen to be empirically a sound method for combining the results of combined axial and transverse loads on the tubes which were so far tested.

## Design Charts

The  $\lambda$ ,  $\sigma$  curves represent for any constant value of Young's modulus, relations between  $\frac{f_b}{R}$ ,  $\frac{f_c}{F_c}$  and  $\sqrt{F_c} \frac{l}{r}$  where  $R$  is the modulus of rupture. From them, combined charts of  $m$ ,  $\lambda$ , and  $\sigma$  have been prepared for materials whose stress-strain curves are affine to these, but combined charts of  $f_b$ ,  $f_c$ , and  $\frac{l}{r}$  or any functions of them alone would differ not only in scale but in shape, for different values of modulus of rupture and  $F_c$ . To convert them into curves representing relations between  $f_b$ ,  $f_c$ , and  $\frac{l}{r}$  it is therefore necessary to assign values to the modulus of rupture and to  $F_c$ .

The modulus of rupture for chrome-molybdenum steel tubing was assumed as 120,000 lb./sq.in., which is 26.3% higher than the 95,000 lb./sq.in. minimum tensile strength prescribed in U.S. Army Air Service Specification No. 10-231-B, June 21, 1926. For duralumin it was assumed as 55,000 lb./sq.in., the minimum tensile strength prescribed in Army Navy Specification AN9092 (1929 issue). These assumptions are based directly on the transverse tests as noted above. Army Navy Specification AN9092 (1929 issue) prescribe for duralumin tubing a minimum tensile yield point\* (determined according to the method described in the specification) of 40,000 lb./sq.in.

If duralumin showed a well-marked yield point such as is

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\*Note.-- The material furnished under this specification has been cold worked after heat treatment. If it is reheated in the process of manufacture, allowance should be made for the resultant decrease in yield point.

found in structural steel, there would be no question that this value would be the proper value to assign to  $F_c$  for duralumin bought under these specifications. As, however, the yield point of duralumin is not well marked, it is necessary to establish a relation between the value of  $F_c$  determined from the column tests and the tensile yield point of the material, determined according to the same procedure as is specified for acceptance tests of the material.

For the material tested this relationship was established as follows: The ratio  $\frac{\text{tensile yield point}}{F_c}$  was calculated from the test data for each size of tubing. These ranged from 1.108 to 1.319, giving 1.231 as the average. The value 1.25 was therefore chosen as a convenient and safe value for this ratio with the experimental errors. Accordingly,  $F_c$  was assumed as 32,000 lb./sq.in. for duralumin tubing bought under these specifications.\*

Because the yield point of the chrome-molybdenum steel which was tested is well marked,  $F_c$  for chrome-molybdenum tubing can safely be assumed equal to the tensile yield point. Since also its ratio to tensile strength is high (average 97%), it would be possible to use safely a higher value for  $F_c$  than 60,000 lb./sq.in. (which was used for this steel) provided the higher yield point was prescribed in the specifications. For

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\*Note.— There is reason to believe that smaller ratios could be used for material not cold worked after heat treatment, but no direct experimental evidence is available.

material bought under the specifications, however, 60,000 lb./sq.in. was assumed.

Using these values of the modulus of rupture  $R$ ,  $f_b = mR$  was computed from  $m$ . With these values of  $F_c$ ,

$$\lambda = \frac{1}{\pi \sqrt{\frac{E}{F_c}}} \frac{l}{r}$$

was computed for values of  $\frac{l}{r}$  in increments of 5 from 30 to 120 and the corresponding values of  $\sigma$  were read from the faired curves (Figures 6 and 7). To obtain these faired curves for intermediate values of  $m$ , auxiliary curves of equal  $\lambda$  were plotted in Figures 8 and 9.  $f_c = F_c \sigma$  was then computed from  $\sigma$ . These values were plotted with  $f_c$  as ordinates and  $f_b$  as abscissas (Figures 10 and 11) and faired curves of equal  $\frac{l}{r}$  values drawn through them. Values of equal  $f_b$  were read from these faired curves. From them were computed  $f_t = f_b + f_c$  and  $\frac{f_b}{f_t}$ . These values located the points on the curves of equal  $\frac{l}{r}$  and equal  $f_b$  in Figures 12 and 13 with  $f_t$  as ordinates and  $\frac{f_b}{f_t}$  as abscissas. On these diagrams the curves of equal  $f_b$  are rectangular hyperbolas.

#### Summary of Present Status of Investigation

1. A semi-empirical method has been found which satisfactorily combines in a single chart the test results on the three sizes of chrome-molybdenum tubing.

2. The same method works satisfactorily on the 1-1/2 in. duralumin tubing, but it is not yet known whether it will work on 1 in. and 2 in. tubing.

3. This method has made it possible safely to raise the design stresses in certain ranges of the diagrams previously worked out by as much as 100% (see Table I).

4. No method has as yet been found which will satisfactorily combine on a single chart both the duralumin and chromemolybdenum tubing so that the conclusions drawn from these cannot safely be extended to other materials with different stress-strain characteristics.

5. However, the consistency with which the experimental points fall close to the faired curves makes it seem probable that this method will prove to be more generally applicable and will be found to rest on a sound theoretical basis.

6. The modulus of rupture of the tubes as determined in these tests is found to be closely correlated with tensile strength but the ratio of modulus of rupture to tensile strength depends in an as yet undetermined way upon the character of the material, the slenderness ratio and the  $\frac{D}{t}$  ratio. For each material the maximum range of variation of the ratio is approximately 20%, between the two materials, duralumin and chromemolybdenum steel; the difference in the average ratio is approx-

imately 30%. If the law of these relationships could be satisfactorily worked out, it would be safe to raise the transverse stresses for some dimensions of tubes by approximately 15%.

7. In the tubing so far tested the maximum  $\frac{D}{t}$  ratio was 47. Up to this value there has been no indication that the maximum loads have been influenced by secondary or detail (crumpling) failure. In the cases in which crumpling has occurred it has never appeared until after the maximum load had been passed. The tests, therefore, give no information concerning the maximum  $\frac{D}{t}$  ratios which can be safely used. It is interesting to note in this connection Robertson's conclusions (based on tests up to  $\frac{D}{t} > 300$ )\* : "That the ordinary strut formulas may be used with confidence for practical calculations on tubular steel struts, provided that the ratio of diameter to thickness of tube wall is less than 100." Tubes of this  $\frac{D}{t}$  ratio are now being used in aircraft construction in England. It would be desirable to be able to determine the limit in the case of combined transverse and axial loading.

8. Since the tests have been confined to circular tubing, it is not known whether they are applicable to other shapes such as streamline or square tubing.

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\*See Southwell, Aircraft Engineering, Vol. I, p.136, 1920. Southwell by mistake says "not less." Robertson actually sets a higher limit.

TABLE I

The reduced values of  $f_c$  for  $\frac{l}{r} = 120$  for  $m = 0$  by  $\lambda, \sigma$  method as compared with the old method\* of reduction of stresses.

M a t e r i a l	$f_c$		Increase of $f_c$ by $\lambda, \sigma$ method per cent
	Old method lb./sq.in.	$\lambda, \sigma$ method lb./sq.in.	
Duralumin 1-1/2 in.	6,780	6,800	0.3
Chrome-molybdenum steel, 1 in.	9,540	20,600	115.8
" 1-1/2 in.	11,140	20,600	85.0
" 2 in.	13,600	20,600	51.4

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Washington, D. C.,  
April 13, 1929.

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\*The old method consisted in reducing all axial stresses in proportion to  $\frac{\text{specification yield point}}{\text{column strength of short specimens}}$ .

## A c k n o w l e d g m e n t

A large part of the chrome-molybdenum tubing tested in this investigation was donated by the Ohio Seamless Tube Company, Shelby, Ohio; the Summerill Tubing Company, Bridgeport, Pa.; and the Delaware Seamless Tube Company, Auburn, Pa. The duralumin tubing was donated by the Aluminum Company of America, Pittsburgh, Pa. Snead and Company, Jersey City, N. J., assisted by heat treating much of the chrome-molybdenum tubing. The generous assistance of these firms contributed greatly to the success of the investigation.



## Legends for Figures

Fig. 1. Relationships between  $\lambda$  and  $\sigma$  for 1 in., 1-1/2 in., and 2 in. chrome-molybdenum steel tubes for different values of  $m$ :

$$\lambda = \frac{\frac{l}{r}}{\pi \sqrt{\frac{E}{F_c}}}$$

$$\sigma = \frac{f_c}{F_c}$$

$$m = \frac{f_b}{R}$$

where  $\frac{l}{r}$  = slenderness ratio,

$E$  = Young's modulus, lb./sq.in.,

$F_c$  = column strength of short specimens in a pure column test, lb./sq.in.

$f_c$  =  $\frac{\text{axial load}}{\text{sectional area}}$ , lb./sq.in.

$f_b$  = bending stress due to transverse load alone, lb./sq.in.

$R$  = modulus of rupture, lb./sq.in.

$m$  is expressed in per cent.

Fig. 2. Relationships between  $\lambda$  and  $\sigma$  for 1-1/2 in. duralumin tubes for different values of  $m$ . For notations, see Figure 1.

Fig. 3. Experimental values of  $f_c$  for 1-1/2 in. duralumin and 1 in., 1-1/2 in. and 2 in. chrome-molybdenum steel tubes in pure column test ( $m = 0$ ).

Fig. 4. Experimental values of  $f_c$  for 1-1/2 in. duralumin and 1 in., 1-1/2 in., and 2 in. chrome-molybdenum steel tubes in combined test for  $m = 20$ . For notations, see Figure 3.

Fig. 5. Experimental values of  $f_c$  for 1-1/2 in. duralumin and 1 in., 1-1/2 in. and 2 in. chrome-molybdenum steel tubes in combined test for  $m = 40, 60$ , and  $80$ . For notations, see Figure 3.

Fig. 6. Faired values of  $\sigma$  for chrome-molybdenum steel tubes.

Fig. 7. Faired values of  $\sigma$  for duralumin tubes.

Fig. 8. Relationships between the faired values of  $\sigma$  and  $m$  for given constant values of  $\lambda$  for chrome-molybdenum steel tubes.

Fig. 9. Relationships between the faired values of  $\sigma$  and  $m$  for given constant values of  $\lambda$  for duralumin tubes.

Fig. 10. Relationships between the reduced values of  $f_c$  and  $f_b$  for 1 in., 1-1/2 in., and 2 in. chrome-molybdenum steel tubes. The stresses obtained from this chart represent the stresses at which chrome-molybdenum steel tubing, complying with U.S. Army Air Service Specification No. 57-180-2, December 8, 1926, may be expected to fail under the corresponding proportions of transverse and axial loads. They contain no allowance for a "factor of safety." The proper "factor of safety" should be provided by the method of design computation used.

Fig. 11. Relationships between the reduced values of  $f_c$  and  $f_b$  for 1-1/2 in. duralumin tubes. The stresses obtained from this chart represent the stresses at which duralumin tubing, complying with Army Navy Specification No. AN9092, 1929 issue, may be expected to fail under the corresponding proportions of transverse and axial loads. They contain no allowance for a "factor of safety." The proper "factor of safety" should be provided by the method of design computation used. The material furnished under this specification has been cold-worked after heat treatment. If it is reheated in the process of manufacture, allowance should be made for the resultant decrease in yield point.

Fig. 12. Relationships between the reduced values of  $f_t$  and  $f_b/f_t$ , where  $f_t = f_c + f_b$  for 1 in., 1-1/2 in., and 2 in. chrome-molybdenum steel tubes. The stresses obtained from this chart represent the stresses at which chrome-molybdenum steel tubing, complying with U.S. Army Air Service Specification No. 57-180-2, December 8, 1926, may be expected to fail under the corresponding proportions of transverse and axial loads. They contain no allowance for a "factor of safety." The proper "factor of safety" should be provided by the method of design computation used.

Fig. 13. Relationships between the reduced values of  $f_t$  and  $f_b/f_t$ , where  $f_t = f_c + f_b$  for 1-1/2 in. duralumin tubes. The stresses obtained from this chart represent the stresses at which duralumin tubing, complying with Army Navy Specification No. AN9092, 1929 issue, may be expected to fail under the corresponding proportions of transverse and axial loads. They contain no allowance for a "factor of safety." The proper "factor of safety" should be provided by the method of design computation used. The material furnished under this specification has been cold-worked after heat treatment. If it is reheated in the process of manufacture, allowance should be made for the resultant decrease in yield point.

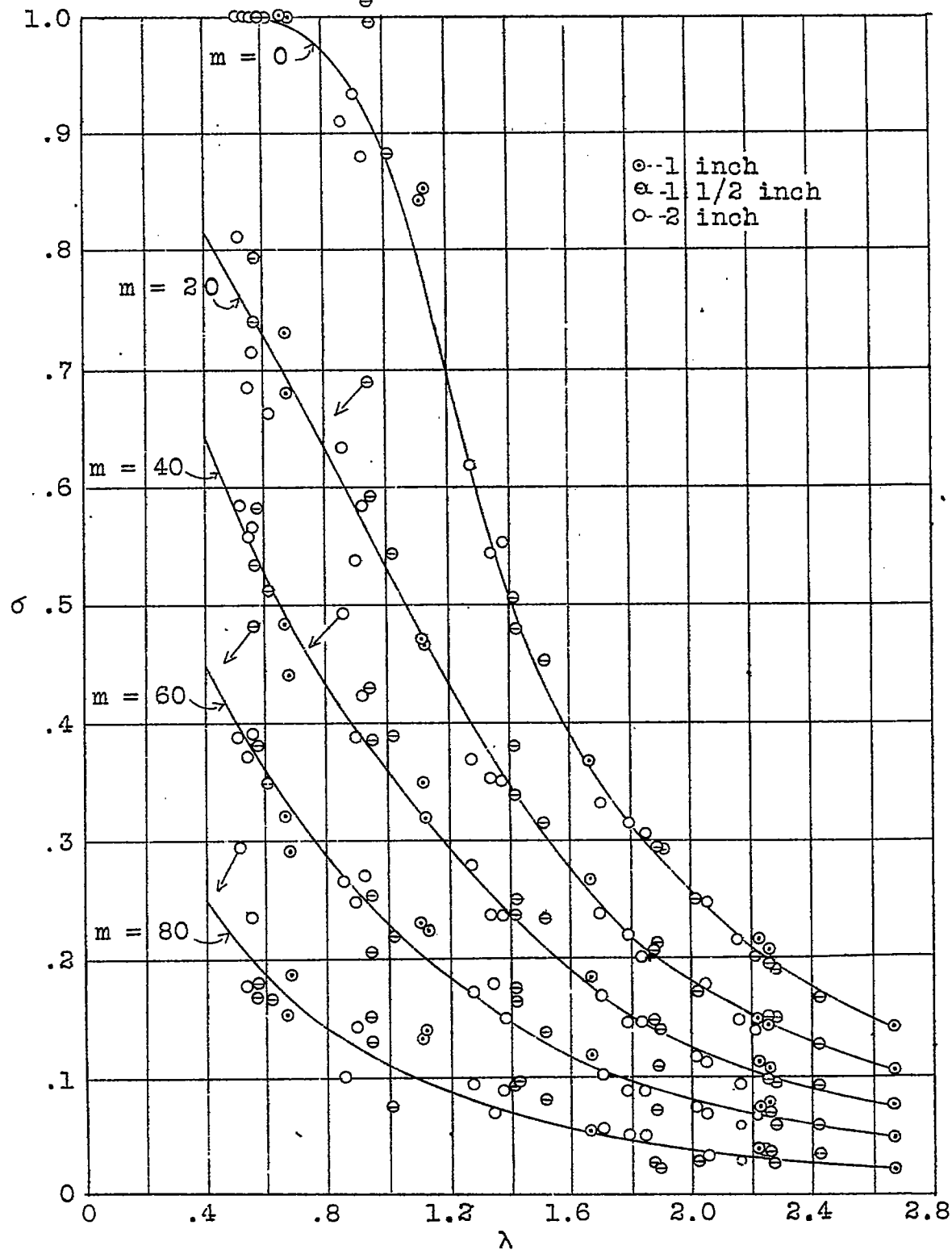


Fig.1

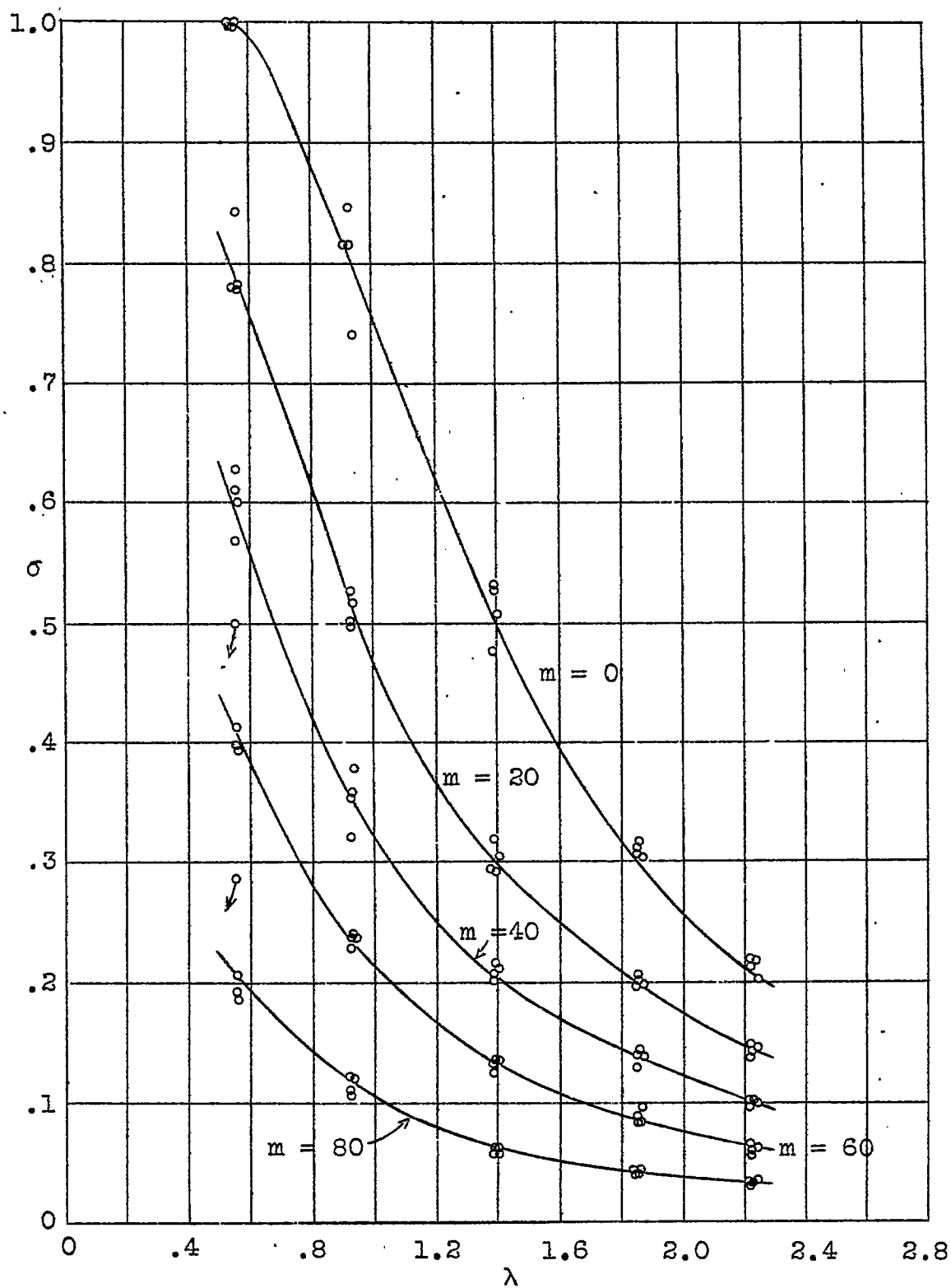


Fig.2

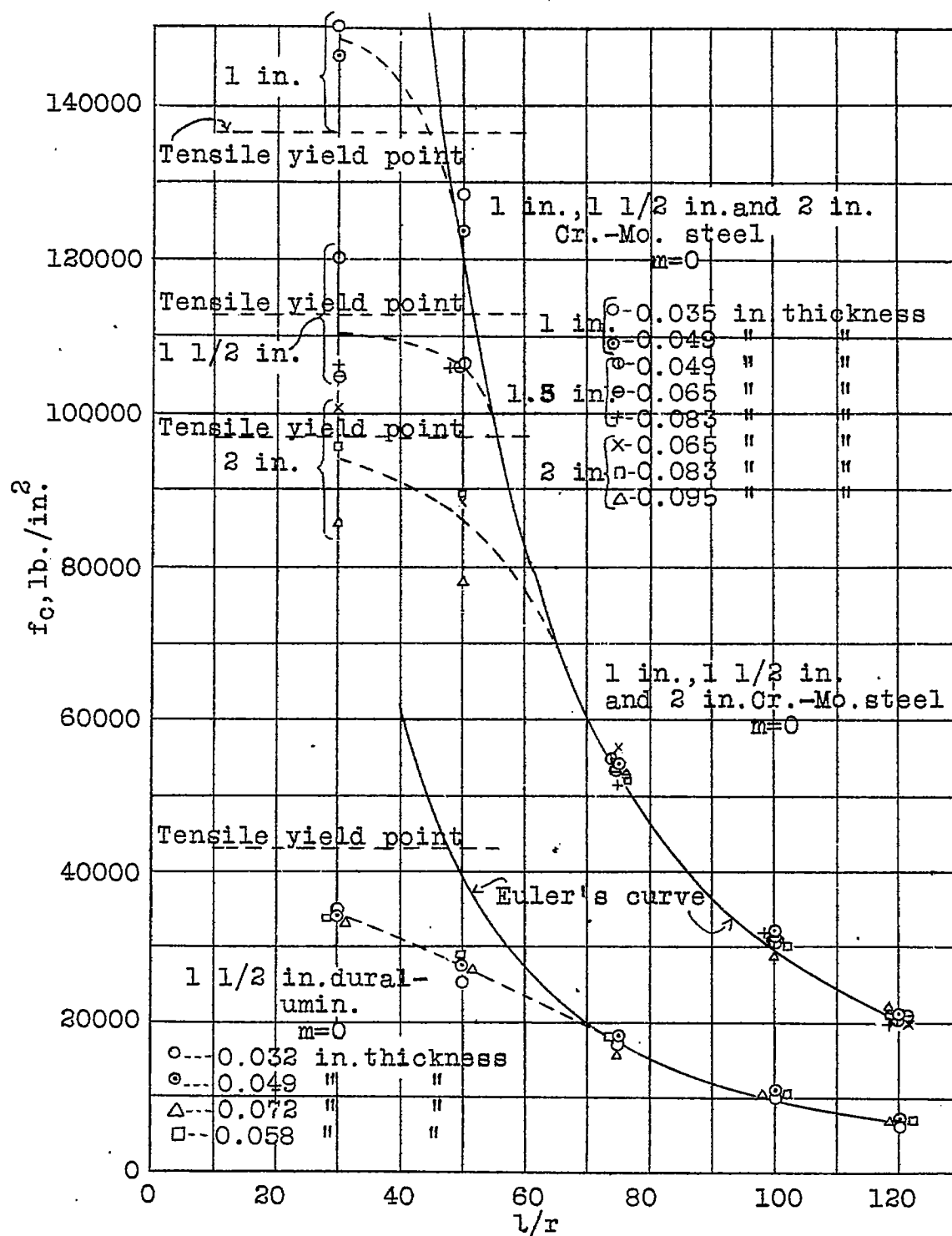


Fig.3

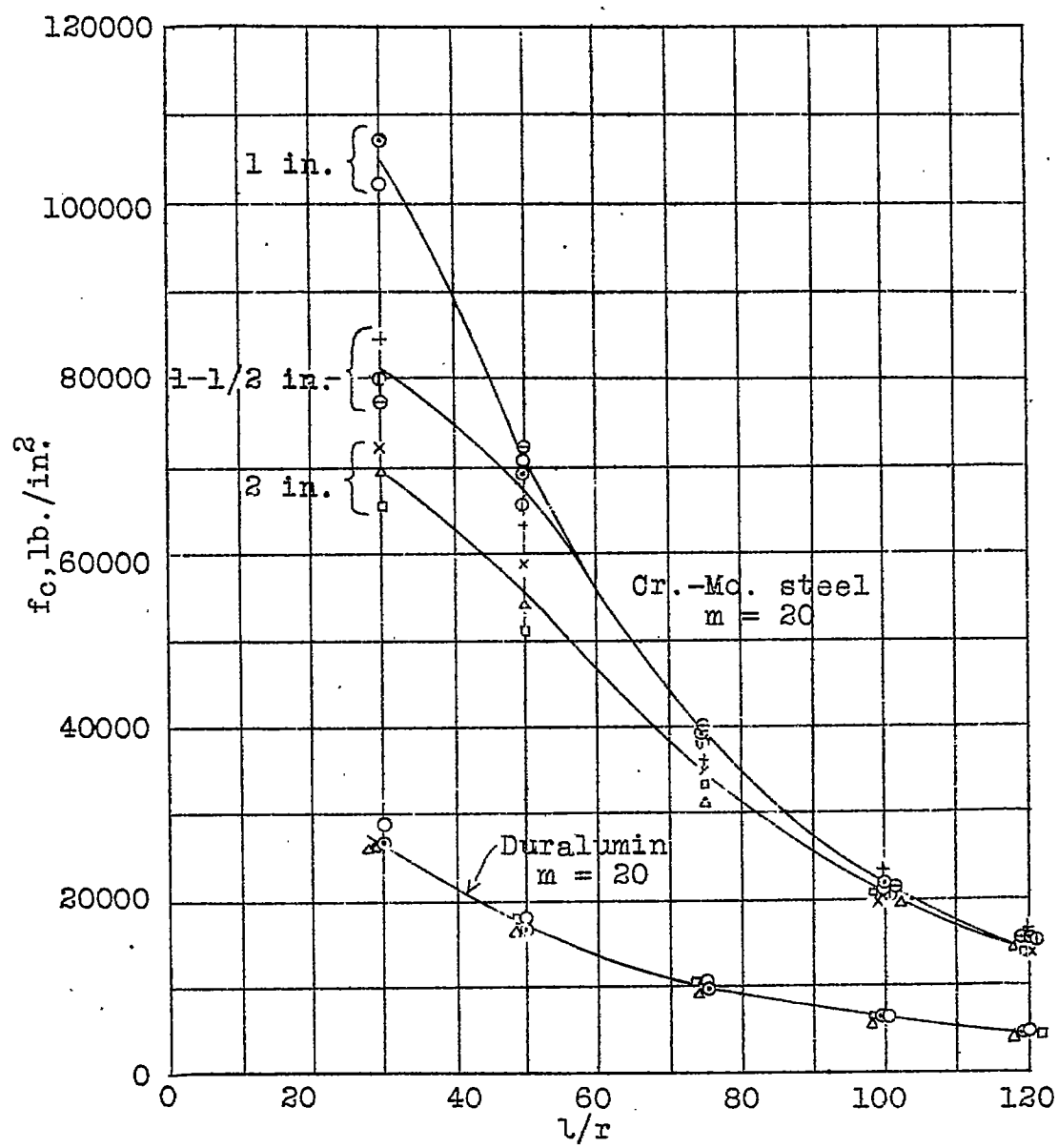


Fig.4

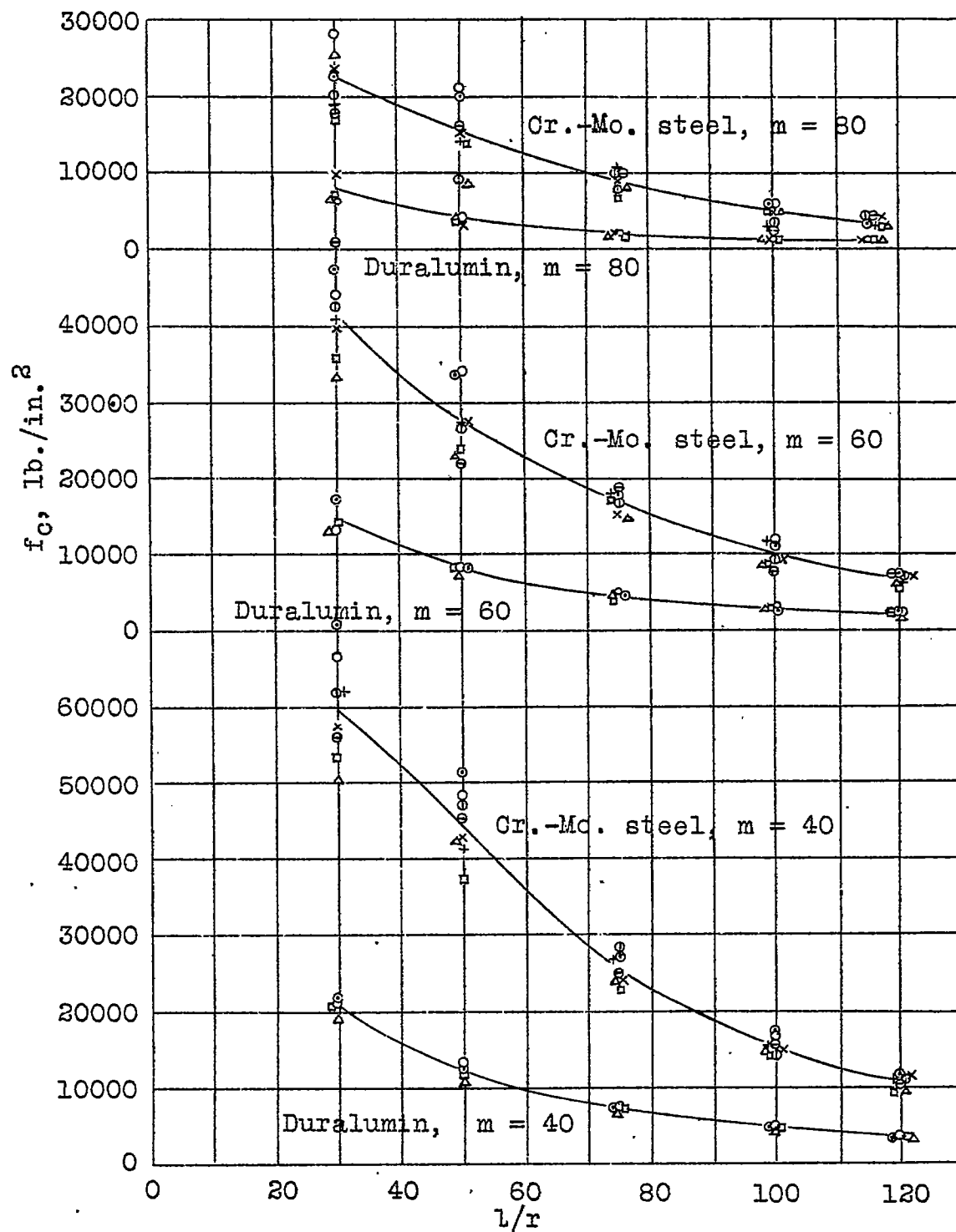


Fig.5



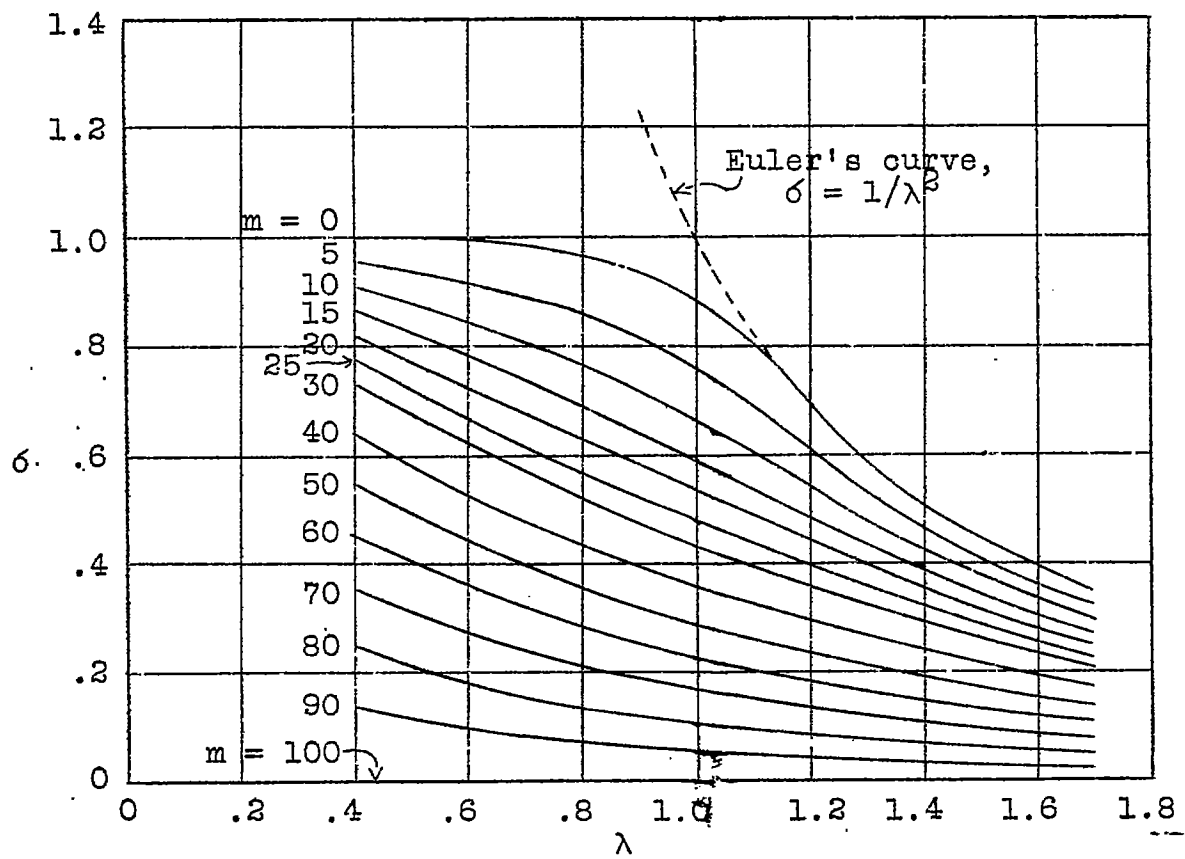


Fig.6

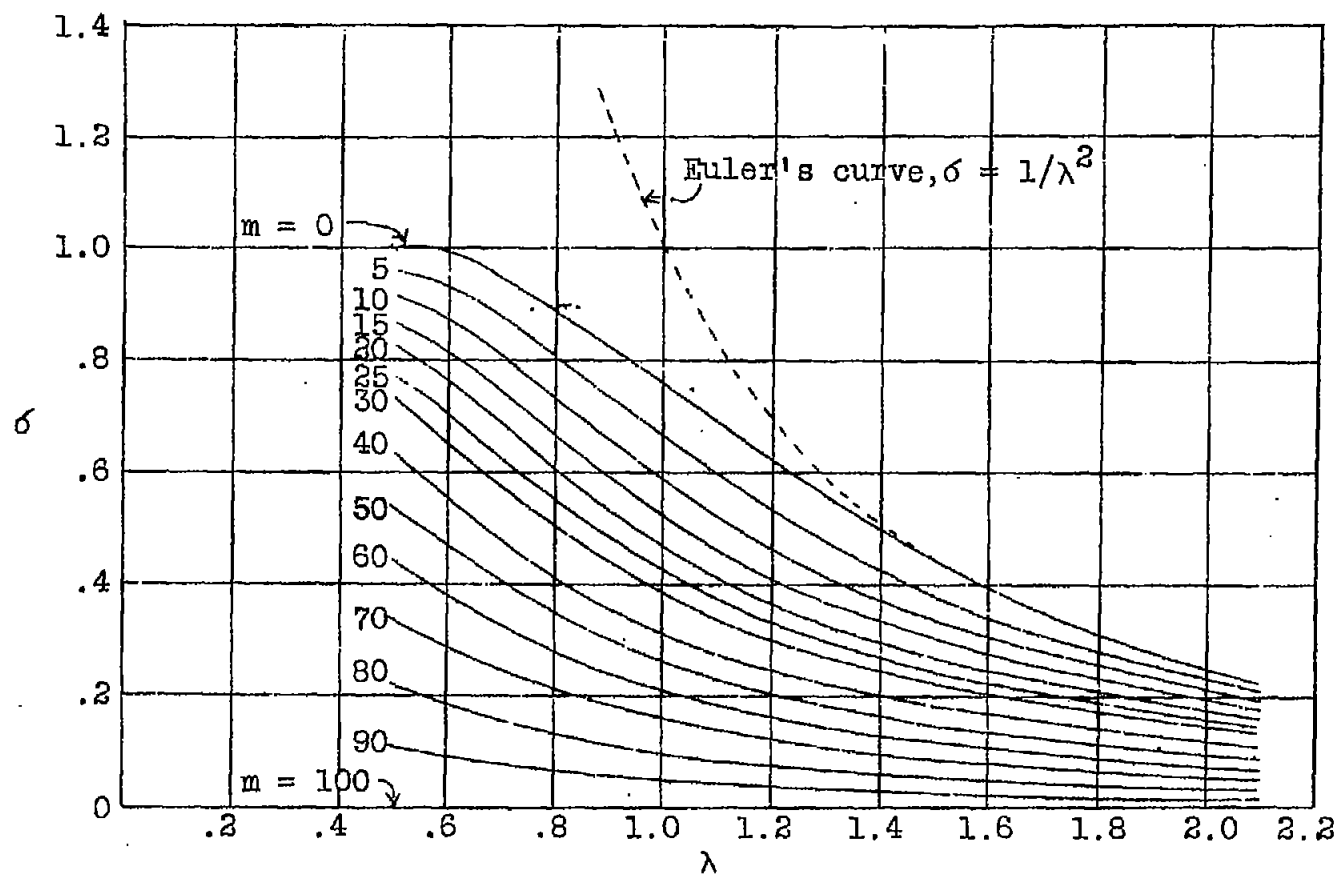


Fig. 7

Fig. 7

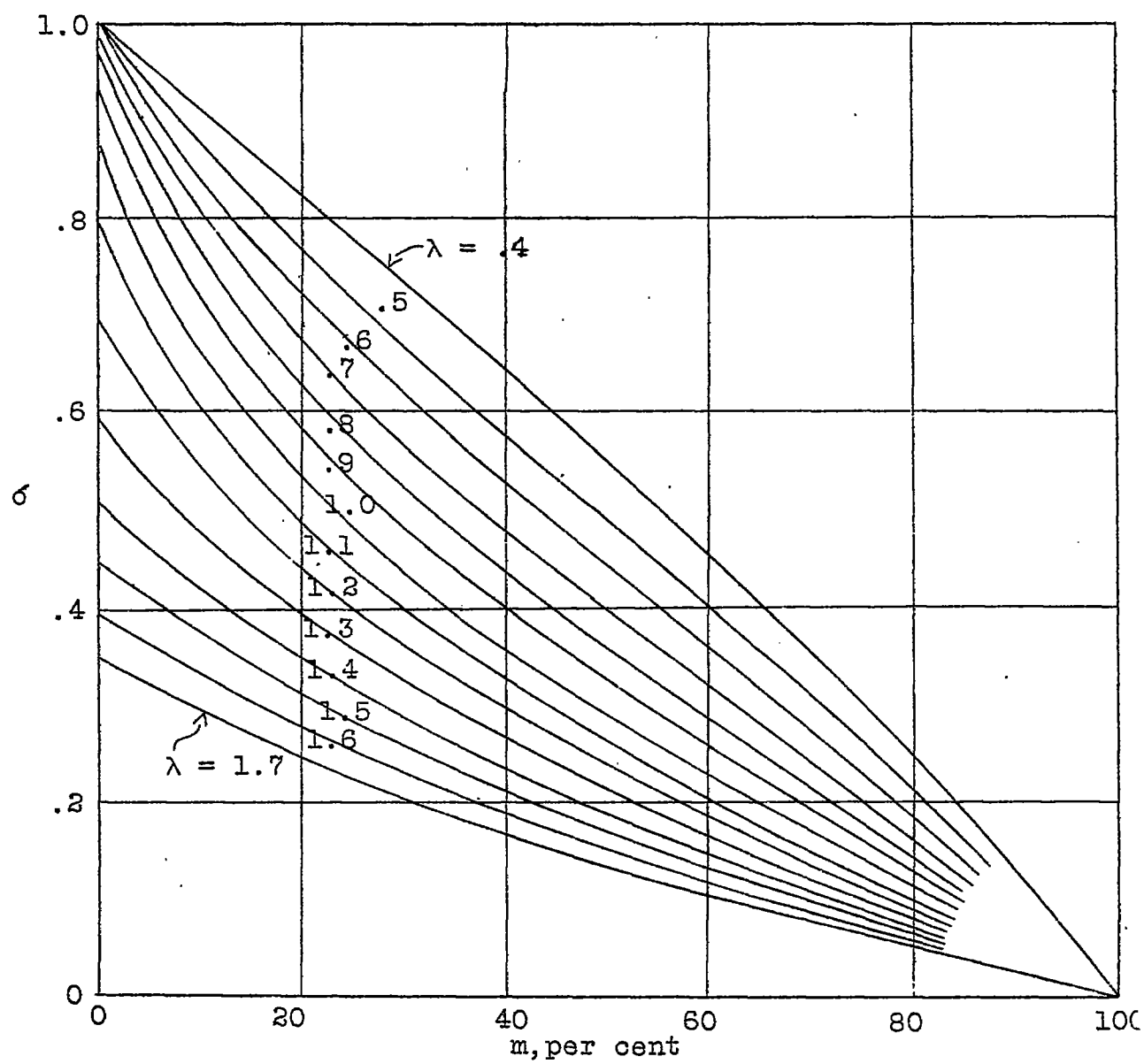


Fig.8

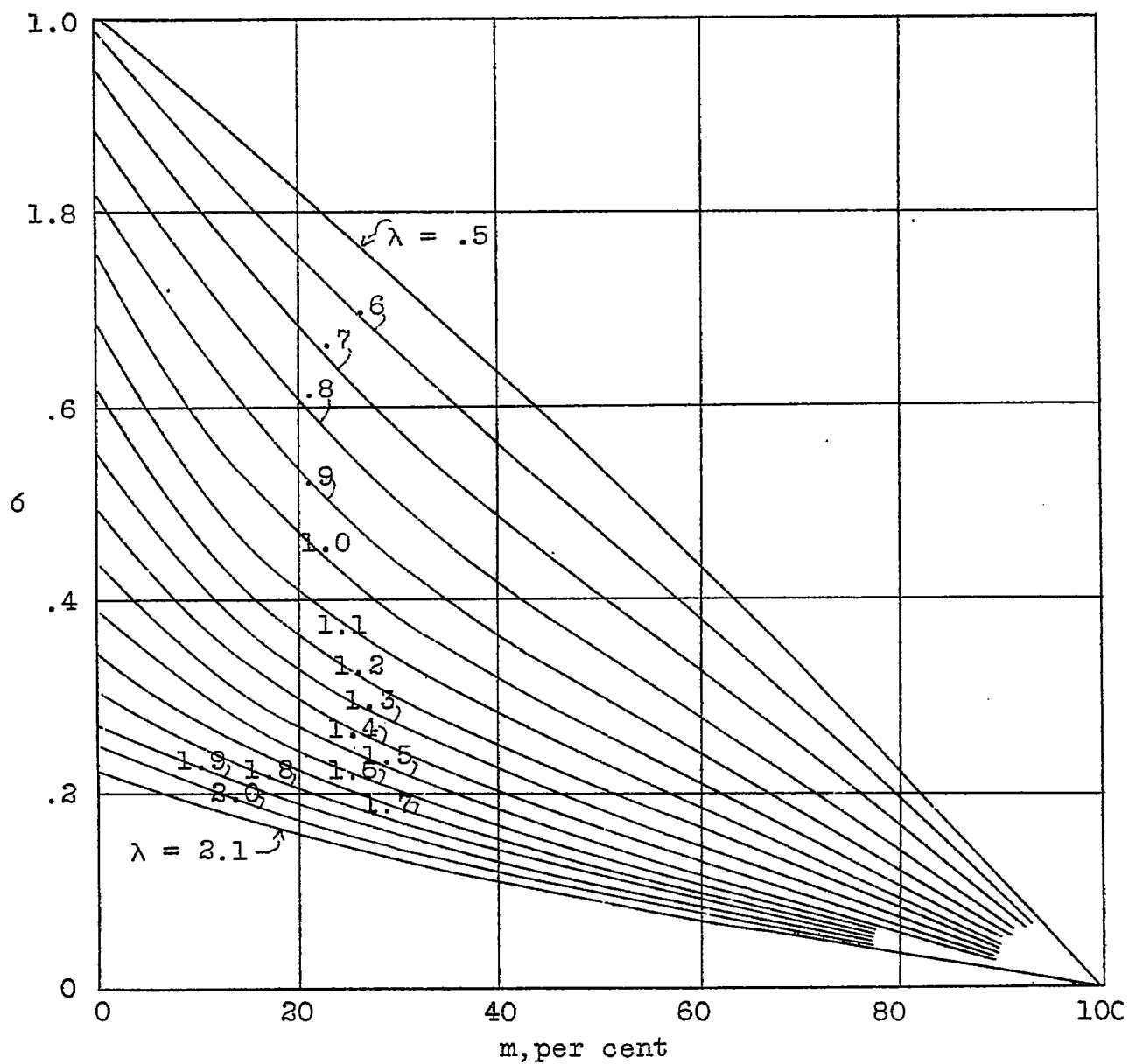


Fig.9

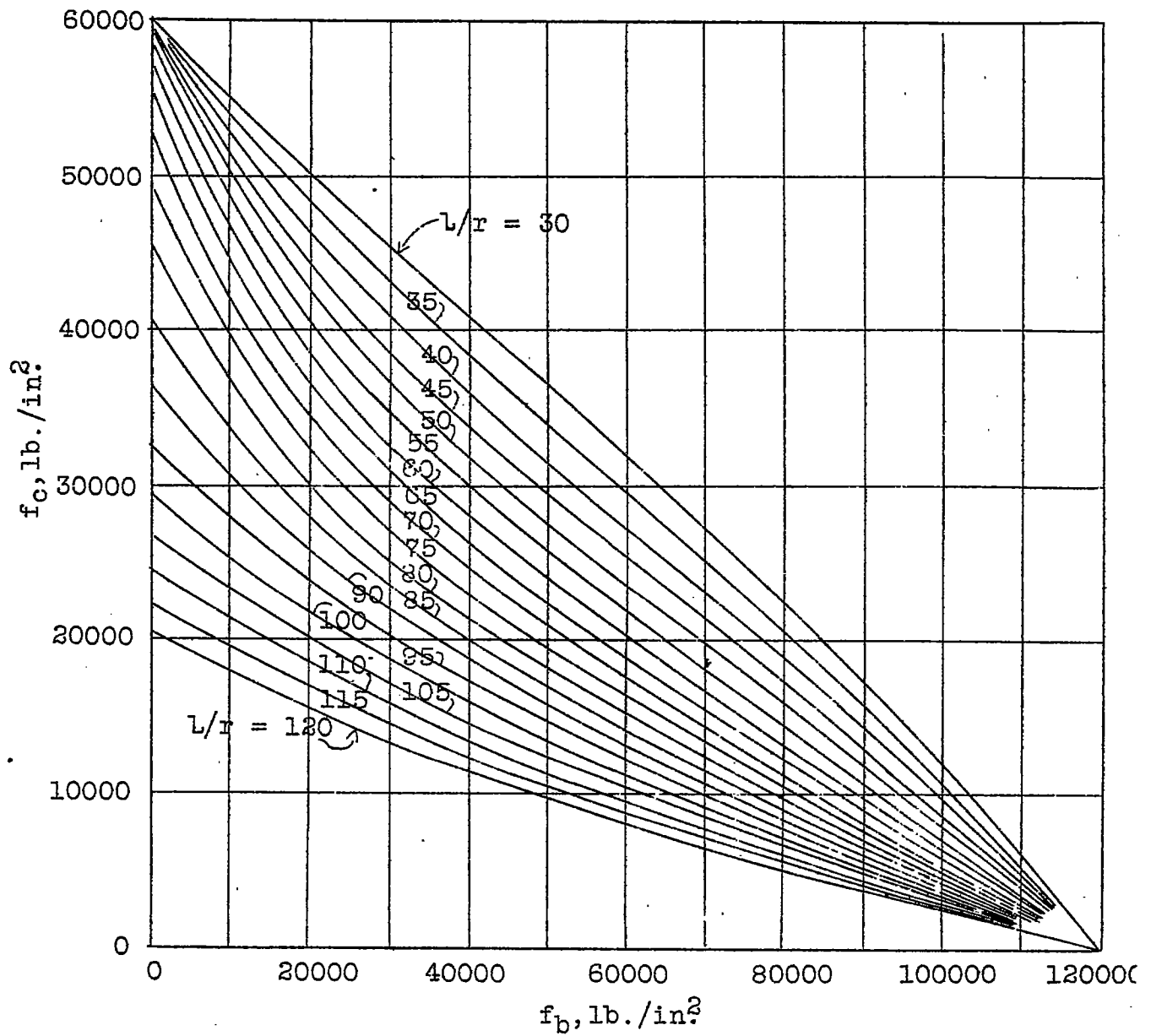


Fig.10

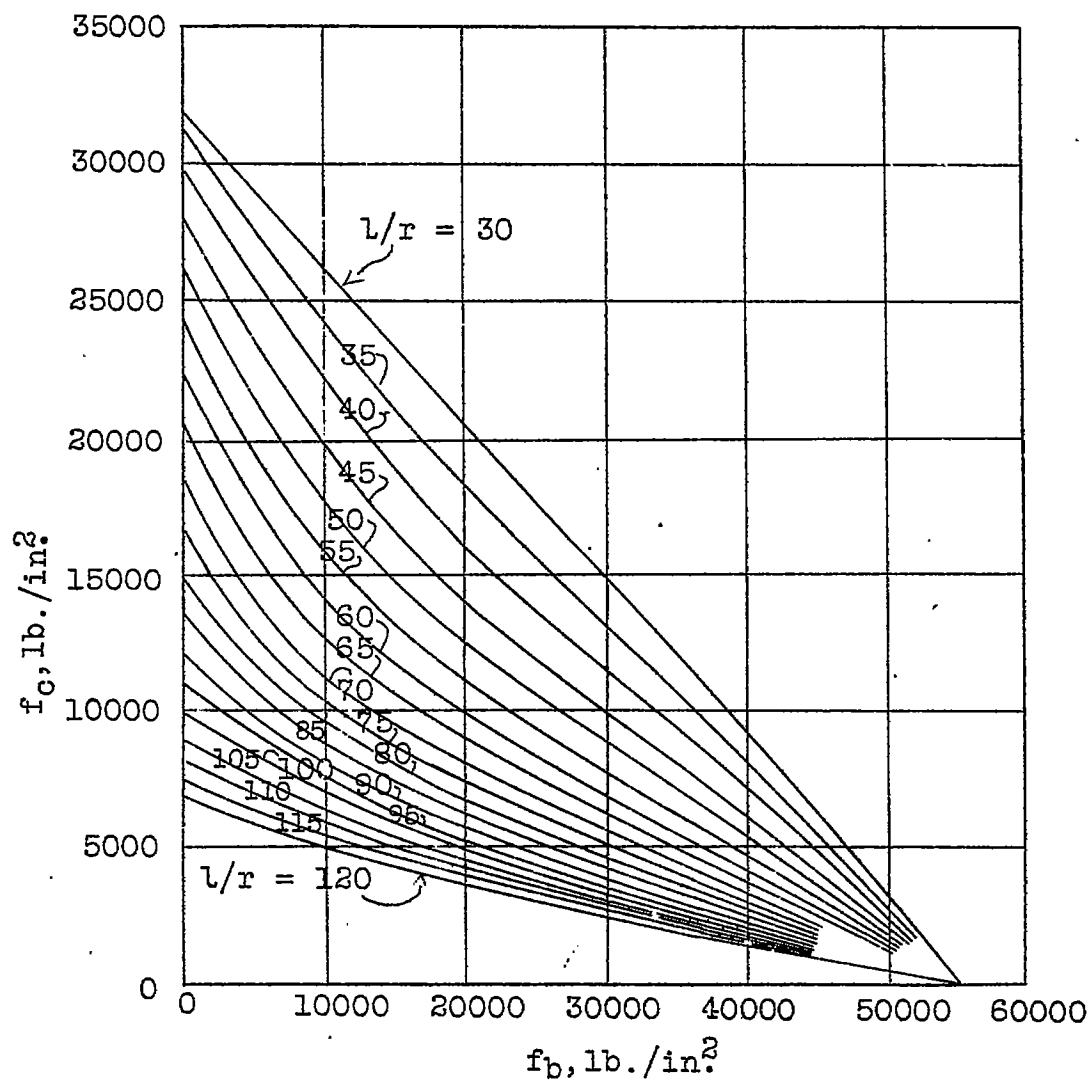


Fig.11

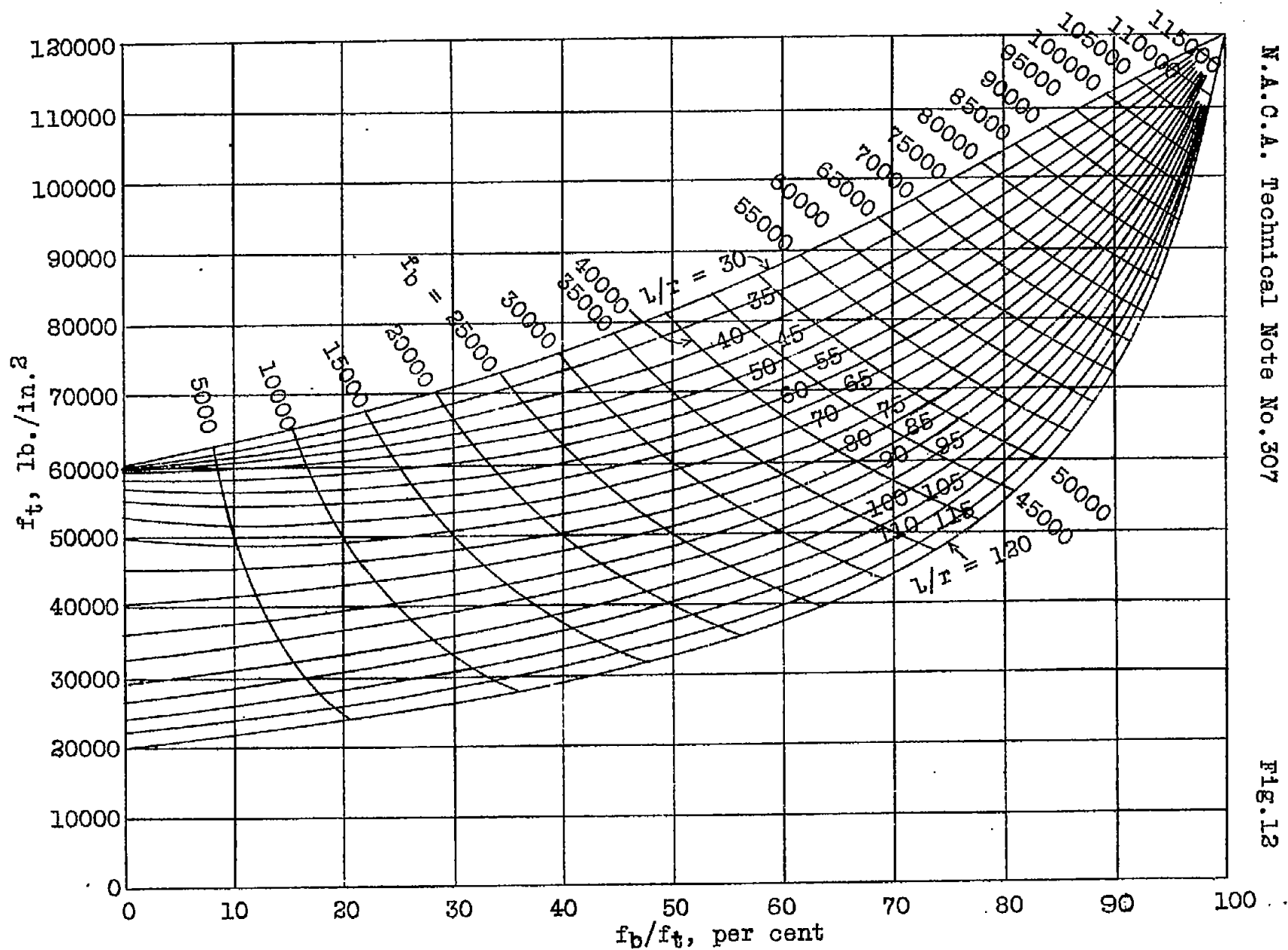


Fig.12

Fig.12

